Conductance oscillations with magnetic field of a two-dimensional electron gas-superconductor junction

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We find the current voltage characteristics of a 2DEG-S interface in magnetic field taking into account the surface roughness. Typically in experiments $L/2R_c \gtrsim 3$, where L is the surface length and R_c is the cyclotron radius. The conductance behaves in experiments usually as $G = g_0 + g_1\cos(2\pi\nu + \delta_1)$; higher harmonics, $g_2\cos(4\pi\nu+\delta_2), \ldots$, are hardly seen. Theories based on the assumption of the interface perfectness can hardly describe qualitatively the visibility of the condctance oscillations and the amplitudes of the harmonics: they predict $g_1 \sim g_2 \sim g_3, \ldots$. Our approach with the surface roughness qualitatively agrees with experiments. It is shown how a disorder at a 2DEG-S interface suppresses the conductance oscillations with ν.

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I. INTRODUCTION

The study of hybrid systems consisting of superconductors (S) in contact with clean 2D normal metals (2DEG) in magnetic field has attracted considerable interest in recent years.[1,2,3] The quantum transport in this type of structures can be investigated in the framework of Andreev refection.[4] When an electron quasiparticle in a normal metal (N) reflects from the interface of the superconductor (S) into a hole, Cooper pair transfers into the superconductor. A number of very interesting phenomena based on Andreev reflection had been studied in the past. For example, if the normal metal is surrounded by superconductors, so we have a SNS junction, a number of Andreev reflections appear at the NS interfaces. In equilibrium this leads to Andreev quasiparticle levels in the normal metal that carry considerable part of the Josephson current; out of the equilibrium, when superconductors are voltage biased, quasiparticles Andreev reflect about $2\Delta/eV$ times transferring large quanta of charge from one superconductor to the other. This effect is called Multiple Andreev Reflection (MAR).[5]

Effect similar to MAR appears at a long enough N-S interface in magnetic field when the magnetic field bends quasiparticle trajectories and makes quasiparticles reflect many times from the superconductor. If phase coherence is maintained interference between electrons and holes can result in periodic, Aharonov–Bohm-like oscillations in the magnetoresistance. The conductance G of a S–2DEG interface in magnetic field was measured in experiments.[6,7,8,9,10] It showed highly nonmnotonic dependence with the magnetic field B [large filling factors were considered]; the most interesting effect was the oscillations of G with the filling factor ν in a somewhat similar manner as in Shubnikov-de Gaas effect.

A phenomenological analytical theory of these phenomena based on an "analogy" with the Aaranov-Bohm effect was suggested in Ref.11. Numerical simulation was made in Ref.12. It was theoretically shown that the transport along the infinitely long S/2DEG interface can be described in the framework of electron and hole edge states.[13] 2DEG-S interfaces investigated in the experiments were not infinitely long, but with the length, L, of the order of few cyclotron orbits, R_c, of an electron in 2DEG at the Fermi energy. Quasiclassical theory of the charge transport through 2DEG-S interface at large filling factors, arbitrary length of the 2DEG-S interface was suggested in Ref.14. Most mentioned above theoretical papers considered the ideal 2DEG-S interface: no roughness. It was shown[14] that when $L \sim 2R_c$, $G(\nu)$ oscillates nearly harmonically with ν, as $\cos(2\pi\nu)$. When $L \sim 4R_c$ harmonics $\cos(4\pi\nu)$ becomes visible and so on... In experiments $L \gtrsim 6R_c$, so one would expect good visibility of $\cos(n\pi\nu)$-harmonics in the conductance, where $n = 1, 2, \ldots$. But if we try to compare theoretically predicted $G(\nu)$ with the experimentally measured one then we find that 1) at $L \gtrsim 6R_c$ only the lowest harmonic $\cos 2\pi\nu$ is seen in the conductance and higher harmonics are absent; 2) the visibility [amplitude] of the conductance oscillations is much smaller than theories predict. The reason of this disagreement is probably the roughness of the 2DEG-S interface in experiments and ideal flatness of this interface in theory.

We try to find in this paper the current voltage characteristics of a 2DEG-S interface in magnetic field taking into account the surface roughness. Our approach with the surface roughness possibly helps to make a step towards explanation of the the experimental results. It is shown that the a disorder at a 2DEG-S interface suppresses high harmonics of the the conductance oscillations with ν.

We consider a junction consisting of a superconductor, 2DEG and a normal conductor segments (see Fig.1). Magnetic field B is applied along z direction, perpendicular to the plain of 2DEG. It is supposed that quasiparticle transport is ballistic (the mean free path of an electron $l_{tr} \gg L$, where L is the length of the 2DEG-S boundary). The current I is supposed to flow between normal (N) and superconducting (S) terminals (the voltage V is applied between them).

Following Ref.15,16,17,18, we shall describe the transport properties of the junction in terms of electron

and hole quasiparticle scattering states, which satisfy Bogoliubov-de Gennes (BdG) equations. Then the current through the 2DEG-S surface is

$$I(V) = \frac{e}{h}\int_0^\infty dE\{f_e\,\mathrm{Tr}[\hat{1} - R_{ee} + R_{he}] - f_h\,\mathrm{Tr}[\hat{1} - R_{hh} + R_{eh}]\}, \quad (1)$$

where

$$f_{e(h)} = \frac{1}{e^{E \mp eV} + 1},$$

V is the voltage of the normal terminal, the energy E is counted from μ of the superconductor, $R_{ee}(E, n_o, n_i)$ is the probability of the (normal) reflection of an electron with the energy E incident on the superconductor in the edge channel with quantum number n_i to an electron going from the superconductor in the channel n_o; the trace is taken in the channel space. Spin degrees of freedom are included into the channel definition.

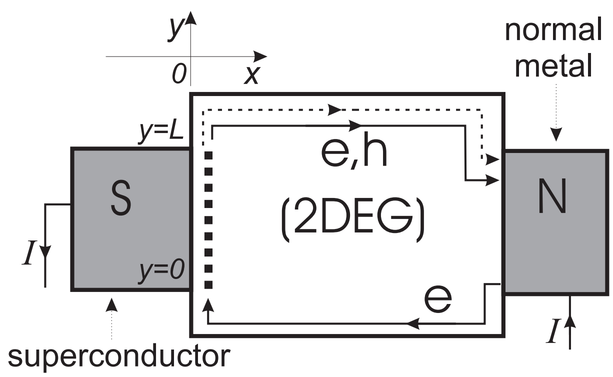

FIG. 1: The device, which we investigate, consists of a superconductor, 2DEG and a normal conductor. An electron injected from the normal conductor in IQH regime goes through an edge state to the superconductor, reflects into a hole and an electron which return to the normal contact through the other edge states.

So the main task of our work is to find the probabilities R_{ee}, *etc.* Then we'll be able to evaluate the current, the conductance, noise and so on.[18] We'll focus on the situation when If $R_c \lesssim L$. Then quasiparticles reflected from the superconductor (S) due to normal and Andreev reflection of the electron return to S again due to bending of the trajectories by magnetic field.

From the first glance it may seem that the reflection probabilities R_{ab}, $a, b = e(h)$ could be be found using the "standard" approach: by matching the incident and outgoing quasiparticle wave functions at $y = 0$ and $y = L$ with the linear combinations of the quasiparticle wave functions at the 2DEG-S boundary corresponding to Andreev edge states.[13] However this procedure does not look efficient at large filling factors [experimental parameter range] and especially for a disordered 2DEG-S interface. Next it is difficult to do the matching in practice because the Andreev bound states wave functions[13] and

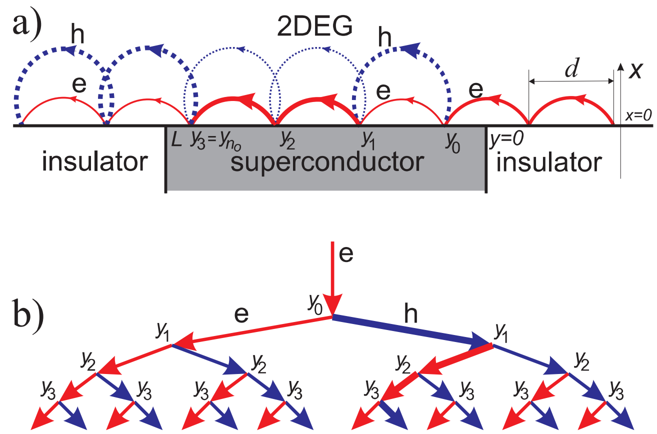

FIG. 2: The propagation of quasiparticles in the quasiclassical approximation can be described in terms of rays. The state of a ray can be found from the equations of classical mechanics. The figures (a-b) show what happens if an electron ray from an edge state of 2DEG comes to the superconductor. The electron ray reflects at $y = y_0$ from the superconductor into electron and hole rays [normal and Andreev reflection]. They reflect in turn at y_1 from the superconductor generating two other electron (red lines) and two other hole rays (blue lines), and so on. To find the probability, e.g., $R_{he}(E, n_o, n_i)$, it is necessary to know the sum of the amplitudes of the eight holes that appear after the last beam-reflection at y_3. Drawing the Fig.2a we assumed that Andreev approximation[4] is applicable [see Eq.21 for conditions] and the 2DEG-S interface is ideally flat.

the wave functions of 2DEG edge states are localized in different domains in $\hat{x}$ direction. The Andreev bound states wave functions penetrate inside the superconductor on the length scale of the order of ξ,[13] but 2DEG edge states wave functions of the incident and outgoing electrons do not penetrate inside the 2DEG edges so deep as ξ.

At large filling factors the quasiclassical approximation is applicable. We show in this paper that within the quasiclassical approximation the matching problem can be solved and R_{ab}, $a, b = e(h)$ explicitly evaluated.

An electron (hole) quasiparticle in 2DEG can be viewed in semiclassics as a beam of rays[20,21] (in a similar way propagation of light is described in optics within eikonal approximation in terms of ray beams[24]). Trajectories of the quasiparticle rays can be found from the equations of classical mechanics. In terms of the wave functions this description means that we somehow make wavepackets from edge states wave functions. Reflection of an electron from the superconductor is schematically shown in Figs.1-5.

II. IDEALLY FLAT 2DEG-S INTERFACE

The transport properties of the ideally flat 2DEG-S interface can be most simply described. The edge channels do not mix at such interface. It means in quasiclassical

language that electrons (holes) skip along the 2DEG edge along the same arc-trajectories, like it is shown in Fig.2. Then $n_o = n_i$ and, e.g., $R_{he}(y_0; n_o, n_i) \propto \delta_{n_o,n_i}$.

The probability of Andreev reflection can be found as follows for the trajectories shown in Fig.2-3:

$$R_{he}(y_0; n_o, n_i) = \delta_{n_o,n_i} \left| e^{i(S_e-\pi/2)} \left\{ r_{he} r_{ee} r_{ee} r_{ee} e^{3iS_e - i3\pi/2 - i\phi(y_3)} + r_{hh} r_{he} r_{ee} r_{ee} e^{iS_h + 2iS_e - i\pi/2 - i\phi(y_2)} + \dots \right\} \right|^2, \quad (2)$$

where r_{ba} is the amplitude of reflection of a quasiparticle (ray) a into a quasiparticle (ray) b from the superconductor; $S_{e(h)}$ is the quasiclassical action of an electron (hole) taken along the part of the trajectory connecting the adjacent points of reflection; $\pm\pi/2$ is the Maslov index[19] of the electron trajectory. The phase $\phi(y)$ arises due to the screening supercurrents. We assume that the superconductor satisfies the description within the London theory [usual in experiments] then $\phi(y) = \phi(0) + \hbar^{-1}\int_0^y d\tilde{y} 2mv_s(\tilde{y})$,[22] where v_s is the superfluid velocity evaluated at $x = 0$ and m is electron mass in the superconductor. We used here the property of London superconductors that the spatial dependence of the vector potential and v_s are small in the perpendicular direction to the superconductor edge on the length scale ξ [on which the wave functions of the scattering electron and hole quasipartciles penetrate in the superconductor].

We introduce the matrix

$$M(y) = \begin{pmatrix} r_{\rm ee}\, e^{i(S_e-\pi/2)} & r_{\rm eh}\, e^{i(S_h+\pi/2)+i\phi(y)} \\ r_{\rm he}\, e^{i(S_e-\pi/2)-i\phi(y)} & r_{\rm hh}\, e^{i(S_h+\pi/2)} \end{pmatrix} \quad (3)$$

that contains the amplitudes of Andreev ($r_{\rm he}, r_{\rm eh}$) and normal ($r_{\rm ee}, r_{\rm hh}$) quasiparticle reflection from the superconductor at the point y. Then the matrix product

$$S^{(3)} = M(y_3)M(y_2)M(y_1)M(y_0), \quad (4)$$

describes the Andreev and normal scattering amplitudes for the case shown in Fig.2,3. So,

$$R_{he}(E, y_0; n_i, n_i) = |S^{(3)}_{21}|^2, \quad (5)$$

$$R_{ee}(E; y_0; n_i, n_i) = |S^{(3)}_{11}|^2. \quad (6)$$

If there are n reflections from the 2DEG-S interface then $3 \to n-1$.

Below we show that the matrix $S^{(n)}$ can be calculated analytically for any integer n when the superfluid velocity $v_s(x = 0, y)$ is constant so the phase $\phi(y)$ is a linear function of y. Then the difference, $\phi(y_n) - \phi(y_{n-1}) = \delta\phi$, does not depend on n because the 2DEG-S interface is flat and $y_n - y_{n-1} = \ldots = y_1 - y_0$. The matrix $M(y_n)$ can be written as

$$M(y_n) = \Phi^\dagger(n) M(y_0) \Phi(n), \quad (7)$$

where

$$\Phi(n) \equiv \begin{pmatrix} \exp\{-\frac{i}{2} n\,\delta\phi\} & 0 \\ 0 & \exp\{\frac{i}{2} n\,\delta\phi\} \end{pmatrix}. \quad (8)$$

Thus

$$S^{(n)} = \Phi^\dagger(n)(M\Phi)^{n+1}\Phi^\dagger, \quad (9)$$

where $M = M(y_0)$, $\Phi = \Phi(1)$.

Then [see Appendix 1]

$$(M\Phi)^{n+1} = \det{}^{(n+1)/2} \cdot \begin{pmatrix} m_{ee}U_n(a) - U_{n-1}(a) & m_{eh}U_n(a) \\ m_{he}U_n(a) & m_{hh}U_n(a) - U_{n-1}(a) \end{pmatrix}, \quad (10)$$

where $\det = \{r_{\rm ee}r_{\rm hh} - r_{\rm eh}r_{\rm he}\}\exp[i(S_e + S_h)]$,

$$a = \frac{r_{\rm ee}e^{i(S_e-\pi/2)-\frac{i}{2}\delta\phi} + r_{\rm hh}e^{i(S_h+\pi/2)+\frac{i}{2}\delta\phi}}{2\sqrt{\det}}, \quad (11)$$

$$m_{ee} = \frac{r_{ee}}{\sqrt{\det}} e^{i(S_e-\pi/2)}, \quad (12)$$

$$m_{eh} = \frac{r_{eh}}{\sqrt{\det}} e^{i(S_h+\pi/2)}, \quad (13)$$

$$m_{he} = \frac{r_{he}}{\sqrt{\det}} e^{i(S_e-\pi/2)}, \quad (14)$$

$$m_{hh} = \frac{r_{hh}}{\sqrt{\det}} e^{i(S_h+\pi/2)}. \quad (15)$$

Eqs.(9)-(15) give an opportunity to find the probabilities R_{ab}, if given the amplitudes of local reflection, r_{ab}. The probabilities R_{ab} depend on the position of the first reflection from the 2DEG-S interface, y_0, that varies in the range $(0, d)$, see Fig.2a. The number of reflections, n, depends on the choice of y_0. So the solution strategy is to calculate the current using Eq.1 with the probabilities R_{ab} defined in Eqs.(9)-(15) and average the result over y_0. The natural choice for the distribution of y_0 is the uniform distribution: $P_{n_i}[y_0] = \theta[d(n_i) - y_0]/2R_c$. So

$$I(V) = \frac{e}{h}\int_0^\infty dE \sum_{n_i} \int dy_0 P_{n_i}(y_0)\{[1 - R_{ee} + R_{he}]f_e - [1 - R_{hh} + R_{eh}]\}f_h. \quad (16)$$

The action $S_e = \int \tilde{\mathbf{k}}_e \cdot d\mathbf{l}$, where $\tilde{k}_e$ is the generalized momentum and the integral is over the quasiparticle trajectory that connects the adjacent points of reflection from the superconductor-2DEG interface.

$$S_e = k_e l_e + \frac{e}{\hbar c}\int \mathbf{A} \cdot d\mathbf{l} = k_e l_e - \frac{|e|}{\hbar c}\Phi_e, \quad (17)$$

$$S_h = -k_h l_h + \frac{|e|}{\hbar c}\Phi_h, \quad (18)$$

where $k_{e(h)} = \sqrt{2m[\mu_{\rm 2DEG} \pm (E + g\mu_B\sigma B)]/\hbar^2}$, $\sigma = \pm 1$, l is the trajectory length and $\Phi_{e(h)}$ is the absolute value of the magnetic field flux through the area bounded by

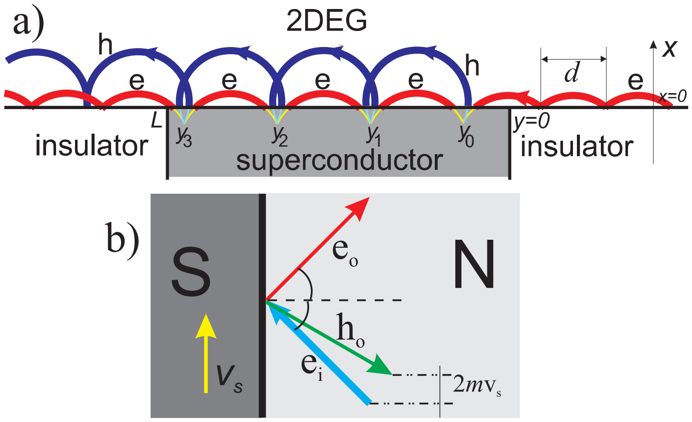

FIG. 3: When the conditions of the Andreev approximation are violated we can not use the assumption that the Andreev-reflected hole velocity is exactly the opposite to the velocity of the incident at the superconductor electron. Then the quasiparticle rays propagate along the 2DEG-S interface as scetched in the figure. The orbits are organized as if the scattering occurs not at the 2DEG-S interface but from the interface in the superconductor lying at some distance from the 2DEG-S interface. This is so because Andreev reflection couples electron and hole orbits with the guiding center x-coordinates $-\delta \pm X$ [see Ref.13], where $\delta \simeq l_B^2 m v_s/\hbar$, $l_B = \sqrt{\hbar c/eB}$ and the value of v_s should be taken at the 2DEG-S interface; for superconductors wider than the London penetration length, $\delta = \lambda_M$. Fig.3b illustrates schematically how Andreev and normal quasiparticle reflections occur from the superconducting interface that carries the supercurrent: i labels the incident quasiparticle, o — the reflected ones.

the quasiparticle trajectory arc and the 2DEG-S interface. The actions can be explicitly written in terms of the filling factor ν and the y-component of the quasiparticle velocity, $v_y^{e(h)}$, at the 2DEG-S interface when $E, g\mu_B\sigma B \ll \mu_{\text{2DEG}}/\nu$:

$$S_{e(h)} = s_{e(h)} \pm \pi(\nu + 1/2), \tag{19}$$

$$s_{e(h)} = 2\left(\nu + \frac{1}{2}\right)\left(\arcsin \Upsilon_{e(h)} - \Upsilon_{e(h)}\sqrt{1-\Upsilon^2_{e(h)}}\right), \tag{20}$$

where $\Upsilon_{e(h)} = v_y^{e(h)}/v^{e(h)}$.

Often the Andreev approximation[4] can be used. Then the Andreev-reflected hole velocity direction may be considered the opposite to the velocity direction of the incident at the superconductor electron [see Fig.2]. The conditions are:

$$v_s \ll v_F^{(\text{2DEG})}, \quad \max(|eV|, T, g\mu_B B) < \Delta < E_F^{(\text{2DEG})}, \tag{21}$$

Then $s_e = s_h$ and

$$S_e - S_h = \frac{|e|}{2\hbar c}\Phi = 2\pi\left(\nu + \frac{1}{2}\right), \tag{22}$$

where Φ is the flux through the Larmor ring-trajectory of an electron in magnetic field B at the Fermi shell.

The problem how to evaluate the r_{ab} amplitudes also simplifies within the parameter range, Eq(21). The conditions mean that the magnetic field could be neglected in the Bogoliubov-de Gennes (BdG) equations [B is already taken into account by the phase ϕ]. Then r_{ab} can be evaluated according to the BTK theory.[15]

When the conditions, Eq.(21), are violated our quasiclassical transport picture in terms of quasiparticle rays can be still applied, see Fig.3. Then the amplitudes r_{ab} are the solutions of the scattering problem for Bogoliubov–de Gennes equations:

$$(E - g\mu_B B)u = \left(\frac{[\mathbf{p} + m\mathbf{v}_s]^2}{2m} - \mu\right)u + \Delta\, v, \tag{23}$$

$$(E - g\mu_B B)v = -\left(\frac{[\mathbf{p} - m\mathbf{v}_s]^2}{2m} - \mu\right)v + \Delta\, u. \tag{24}$$

Here m, g and μ should be considered different in S and 2DEG. The spatial distribution of the superfluid velocity is fixed by the London equation, $\text{rot}\, m\mathbf{v}_s = -\mathbf{B}\, e/c$. Usually there is a barrier at the 2DEG-S interface, we did not write its contribution to BdG explicitly.

It follows from Eq.(16) that at zero temperature and voltage the conductance is:

$$G = \frac{2e^2}{h}\sum_{n_i}\sum_s P_s \frac{|r_{eh}|^2 \sin^2[s \arccos(\sqrt{|r_{ee}|^2}\cos(\Omega))]}{1 - |r_{ee}|^2\cos^2(\Omega)}, \tag{25}$$

where [we remind again] spin is included at the definition of the channel index, $\Omega = (S_e - S_h - \pi)/2 + \theta - \delta\phi/2$; $\theta = \arg(r_{ee})$ is the phase of the amplitude of electron – electron reflection from the superconductor. If the superconductor characteristic dimensions are larger than λ_M – the Meissner penetration length of the superconductor then $\delta\phi/2 = 2\lambda_M k_\perp$, where $k_\perp = k_\perp(n_i)$ is the perpendicular component the quasiparticle momentum when it reflects from the superconductor. The function P_s is the probability that the orbit describes s reflections from the surface of the superconductor; this function originates from the averaging over y_0 discussed above. P_s can be expressed through the maximum number of jumps, $g_m = [L/d]$, over the S-2DEG surface with the length L, where $[\ldots]$ denotes the integer part:

$$P_s = \begin{cases} \frac{L - g_m d}{d} & \text{if } s = g_m + 1, \\ 1 - \frac{L - g_m d}{d} & \text{if } s = g_m, \\ 0 & \text{otherwise.} \end{cases} \tag{26}$$

The conductance, Eq.(25) [as well as the current, Eq.(16)] is an oscillating function of ν:

$$G(\nu) = \sum_{n=0}^{\infty} g_n \cos(2\pi\nu n + \delta_n), \tag{27}$$

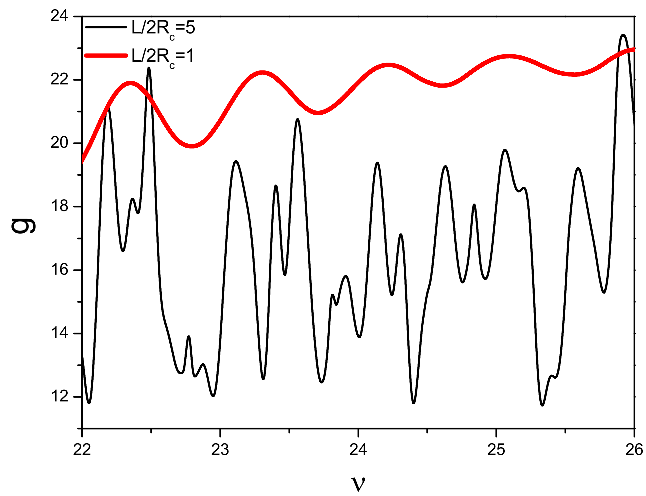

FIG. 4: The (dimensionless) zero bias conductance oscillations with ν. The parameters are the following: $k_F^{(2DEG)} = 2 \cdot 10^6\,\mathrm{cm}^{-1}$, $\delta\phi/4 = 20$ [corresponds to NbN film with the width of the order of 100nm]. The $L = 3\,\mu m$ [$L/2R_c \simeq 5$ at $\nu = 25$] for the lower curve and $L = 0.6\,\mu m$ [$L/2R_c \simeq 1$ at $\nu = 25$] for the upper (thick) curve. We neglected the Zeeman splitting (typically small). The 2DEG-S interface scattering amplitudes, r_{ab}, were taken according to the BTK model[15] with $Z = 0.6$. The curves were made using Eq.(25).

where g_n are the Fourier coefficients and δ_n – the "phase shifts". When the length of the interface, $L \lesssim 2R_c$, then the leading contribution to the conductance (current) gives the zero harmonic; while $2R_c \lesssim L \lesssim 4R_c$ then $G \approx g_0 + g_1 \cos(2\pi\nu + \delta_1)$; when $4R_c \lesssim L \lesssim 6R_c$ the second harmonics becomes relevant, and so on...

How the conductance changes with ν is illustrated in Fig.4. Typically in experiments $L/2R_c \gtrsim 3$ [thin black curve]. But the conductance behaves in experiments as if $G = g_0 + g_1 \cos(2\pi\nu + \delta_1)$; higher harmonics, $g_3, \ldots$, are not seen. But our theory based on the assumption of the interface flatness predicts $g_1 \sim g_2 \sim g_3$ while $L/2R_c \gtrsim 3$, see Fig.4. The reason of the discrepancy between our theory and the experiment is the assumption that the 2DEG-S interface is ideally flat. Disorder at the 2DEG-S interface makes $g_0 > g_1 > g_2, \ldots$. Below we demonstrate it.

III. DISORDERED 2DEG-S INTERFACE

Usually 2DEG-S interface is not ideally flat. The disorder at the interface can be divided at two classes long range and short range with the respect to the characteristic wavelength, λ_F, in 2DEG. Presence of the long range disorder implies that the 2DEG-S interface position fluctuates around the line $x = 0$ at length scales much larger than the $\lambda_F^{(2DEG)} \sim 10^{-6}\mu m$. Photographs of the experimental setups do not allow to think that 2DEG-S interface bends strongly from the line $x = 0$. So this kind of the disorder is likely not very important.

The short-range disorder includes the fluctuations of the surface at length scales smaller than $\lambda_F^{(2DEG)}$; impurities, clusters of atoms at the surface due to defects of the lithography and so on... When, for example an electron ray falls on the disordered 2DEG-S surface the reflected electron rays go off the surface not at a fixed angle but they may go at any angle with certain disorder induced probability distribution. The phases that carry the reflected electron rays going off the surface at different angles may be considered random, so the reflected electron rays can be considered as incoherent.[26] But to any reflected electron ray an Andreev reflected hole ray is attached that is coherent with the electron. So the interference of rays [that produces the conductance oscillations] may be not killed completely by the short range disorder. Below it will be clarified.

"Weak" short range disorder at 2DEG-S interface does not destroy the Andreev edge states but it induces transitions between the edge states, see Fig.5b. Andreev edge states in quasiclassics fix electron-hole orbit-arcs with the same beginning and end. The quasiclassical picture of the disorder-induced transitions is shown in Fig.5a.

It is shown below how to describe the transport properties of the weakly short-range disordered 2DEG-S interface in magnetic field. We assume that the Andreev approximation conditions, Eq.(21), are fulfilled [general case brings to qualitatively similar results for the current, it requires same idea of calculations but it is more cumbersome] then $s_e = s_h$ and the fluctuating quantity is $\delta\phi$. Similarly as before we introduce the matrix

$$M(y_n) = \Phi^\dagger(n) M \Phi(n)\, e^{i\sum_{i=0}^{n} s_i}, \tag{28}$$

where s is defined in Eq.(20),

$$\Phi(n) \equiv \begin{pmatrix} \exp\{-\frac{i}{2}\sum_{i=0}^{n}\delta\phi_i\} & 0 \\ 0 & \exp\{\frac{i}{2}\sum_{i=0}^{n}\delta\phi_i\} \end{pmatrix}, \tag{29}$$

and

$$M = \begin{pmatrix} r_{\mathrm{ee}}\, e^{i\pi\nu} & r_{\mathrm{eh}}\, e^{-i\pi\nu} \\ r_{\mathrm{he}}\, e^{i\pi\nu} & r_{\mathrm{hh}}\, e^{-i\pi\nu} \end{pmatrix}. \tag{30}$$

As before, for the situation sketched in Fig.5a, $S^{(3)} = M(y_3)M(y_2)M(y_1)M(y_0)\, e^{i\sum_{i=0}^{3} s_i}$, and, e.g., $R_{he} = |S_{21}^{(3)}|^2$. It is clear that $e^{i\sum_{i=0}^{3} s_i}$ does not influence on the probabilities R_{ab} so we'll omit this term below. In general case:

$$S^{(n)} = \Phi^\dagger(n)[M\Phi_n \ldots M\Phi_1 M\Phi_0]\Phi_0^\dagger. \tag{31}$$

here

$$\Phi_n \equiv \begin{pmatrix} \exp\{-\frac{i}{2}\delta\phi_n\} & 0 \\ 0 & \exp\{\frac{i}{2}\delta\phi_n\} \end{pmatrix}, \tag{32}$$

where $\delta\phi_n = \phi_n - \phi_{n-1}$ and $\delta\phi_0 \equiv 0$.

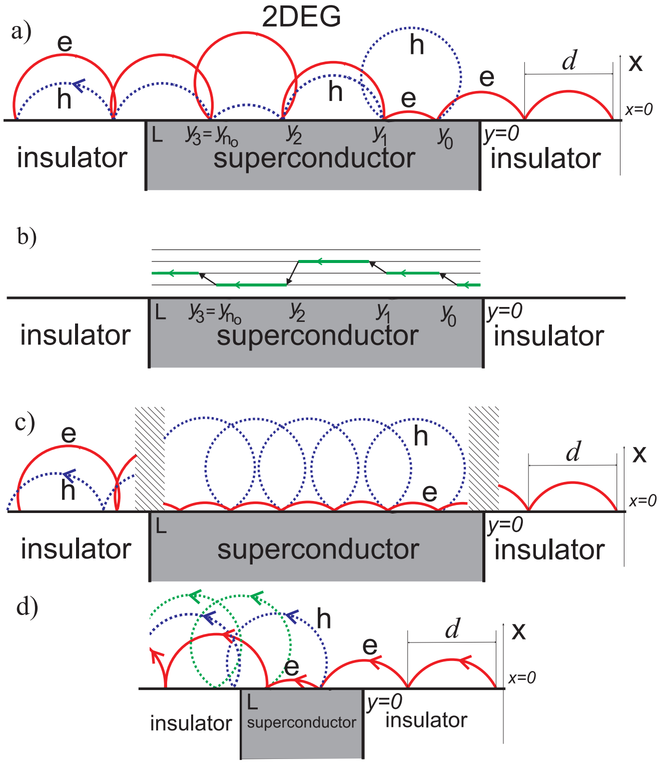

FIG. 5: Relatively weak short-range disorder at 2DEG-S interface induces transitions between the Andreev edge states, see Fig.5b. Andreev edge states in quasiclassics make electron-hole orbit-arcs with the same beginning and end. The quasiclassical picture of the disorder-induced transitions is shown in Fig.5a. Disorder at the edges of the 2DEG-S interface leads to the orbits depicted in Fig.5c. Strong disorder destroys Andreev edge states; there are no oscillations in the conductance because quasiparticles reflected from the strongly disordered 2DEG-S surface are incoherent; orbits are shown in Fig.5d.

The probabilities can be found in the following way:

$$R_{ee} - R_{he} = \frac{1}{2}\,\mathrm{Tr}\,\{\sigma_z M \Phi_n M \Phi_{n-1} \dots \\ \times\ \Phi_2 M \Phi_1 M \sigma_z M^\dagger \Phi_1^\dagger M^\dagger \Phi_2^\dagger \dots \Phi_{n-1}^\dagger M^\dagger \Phi_n^\dagger M^\dagger \Big\}. \quad (33)$$

Here we neglected the term $\Phi^\dagger(n)$ that enters Eq.(31) because it does not contribute to the probabilities R_{ab}. If one wants to find R_{ee} then σ_z should be substituted by $(\sigma_0 + \sigma_z)/2$ in the last equation. If R_{eh} is wanted then the first σ_z should be substituted by $(\sigma_0 + \sigma_z)/2$, the second – by $(\sigma_0 - \sigma_z)/2$.

As in the previous section we should average the current over y_0. This operation is closely related to the disorder averaging because shifting y_0 we'll make the trajectories go through different disorder realizations. The phase jumps $\delta\phi_n$ fluctuate due to the disorder.

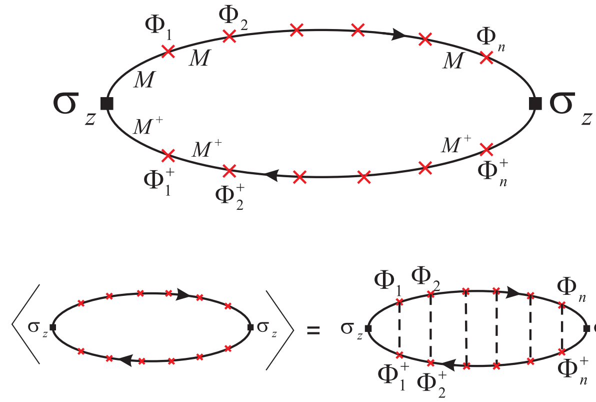

FIG. 6: Disorder averaging of the transmission probabilities. The loop corresponds to the trace in Eq.(33). The $\times$-vertices are $\Phi, \Phi^\dagger$, the solid lines represent $M, M^\dagger$, the dashed lines are the $\langle \Phi\Phi^\dagger\rangle$ correlators.

The disorder average

$$\langle [\Phi_a]_{ij} [\Phi_b]^\dagger_{pq} \rangle = \delta_{ab}\delta_{ij}\delta_{pq}(\delta_{ip} + \Lambda_{ip}), \quad (34)$$

$$\Lambda_{ip} = \begin{pmatrix} 0 & \langle e^{-i\delta\phi_a}\rangle \\ \langle e^{i\delta\phi_a}\rangle & 0 \end{pmatrix}. \quad (35)$$

The averages like $\langle (1-\delta_{ab}) e^{-i(\delta\phi_a \pm \delta\phi_b)/2}\rangle = 0$ because quasiparticles acquire a random phase reflecting from the disordered interface as it was mentioned above.

$R_{ee} - R_{he}$ can be treated perturbatively over small Λ. The difference $R_{ee} - R_{he}$ can be treated perturbatively over Λ. It is natural to assume that $\delta\phi_n$ are gaussian distributed:

$$\langle \delta\phi_a \rangle = \overline{\delta\phi}, \quad (36)$$

$$\langle\langle \delta\phi_a\, \delta\phi_b \rangle\rangle = 2\eta\delta_{ab}, \quad (37)$$

where $\langle\langle \dots \rangle\rangle$ means the irreducible average. Then

$$\langle e^{i\delta\phi_a}\rangle = e^{i\,\overline{\delta\phi}}\, e^{-\eta}. \quad (38)$$

While $\eta \ll 1$, weak fluctuations, the results of the previous section are valid. The opposite case we discuss below.

Lets $\eta \gg 1$. In the zero order over Λ, for $n = 3$, the average probabilities

$$R_{ee} - R_{he} = \\ \mathrm{Tr}\left\{ (\sigma_z)_{i_2 i_2} |M_{i_2 i_3}|^2 |M_{i_3 i_4}|^2 |M_{i_4 i_5}|^2 |M_{i_5 i_6}|^2 (\sigma_z)_{i_6 i_6} \right\}. \quad (39)$$

Thus we see that in the "completely incoherent case" [zero order over Λ] the average probabilities can be found as the corresponding elements of the matrix $\tilde{S}^n$:

$$\tilde{S}^n = \begin{pmatrix} |r_{ee}|^2 & |r_{eh}|^2 \\ |r_{he}|^2 & |r_{hh}|^2 \end{pmatrix}^n, \quad (40)$$

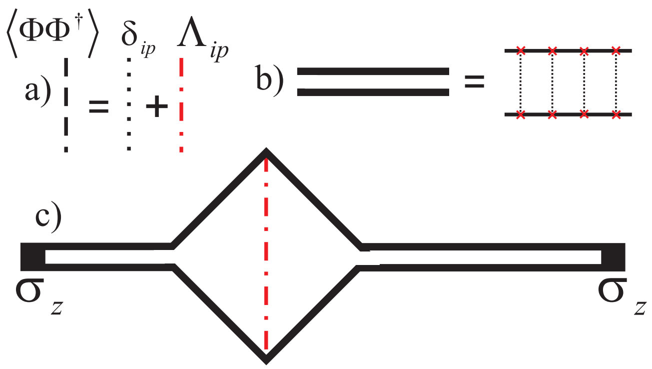

FIG. 7: Typical contribution to the average of the bubble in the first order over Λ is schematically shown in the figure, Fig.7c. The $\langle\Phi\Phi^\dagger\rangle$ line splits into the sum of two: the first one curresponds to the δ_{ip}-term in Eq.(34) and the second one – to the Λ-term, Fig.7a. The parallel lines show the part of the bubble, Eq.(33), where the first order of Λ is taken ("diffusion"), it does not carry interference information, Fig.7b. The diamond means the place where Λ-line is inserted; this diamond provides oscillations of the current with ν. One should sum all n diagram like shown in Fig.7 [they differ by the position of the diamond] to find the average of the bubble in the first order over Λ.

for example $R_{ee} = [\tilde{S}^n]_{11}$. Explicit form of $\tilde{S}^n$ can be easily found using the theorem mentioned in the Appendix A. More interesting is the first order expansion of the probabilities R_{ab} over Λ. After calculations diagrammatically illustrated in Fig.7 (see Appendix B) we find for the average probabilities

$$A_n = R_{ee} - R_{he} = \\ = R^n \left[1 + e^{-\eta} \frac{4(n-1)|r_{ee}|^2|r_{eh}|^2}{R^2} \cos(2\Omega)\right] \quad (41)$$

where $\Omega = \pi\nu + \theta_{ee} - \overline{\delta\phi}/2$, $R = |r_{ee}|^2 - |r_{eh}|^2$ and n is the number of reflections from the 2DEG-S interface. All trajectory-dependent quantities that enter Eq.(41) should be evaluated for the trajectory with equal electron and hole arcs [$\Upsilon_{e(h)} = 0$].

So, finally the zero-bias conductance is

$$G = \frac{4e^2}{h}\nu \sum_n P_n\,[1 - A_n], \quad (42)$$

where P_s is defined in Eq.(26), but with $d \to 2R_c$. The conductance oscillations according to Eq.(42) are illustrated in Fig.8.

The result similar to Eqs.(41),(42) would be obtain calculating the influence of the disorder at the edges of 2DEG-S interface, Fig.5c, on the magneto-conductance.

IV. CONCLUSIONS

We found in this paper the current voltage characteristics of a 2DEG-S interface in magnetic field taking into account the surface roughness. Our approach with the surface roughness possibly removes the contradiction between the theory and the experiment. It is shown that the a disorder at a 2DEG-S interface suppresses the conductance oscillations with ν. The measurement of the magneto-conductance of a 2DEG-S boundary is the test for the degree of the boundary roughness.

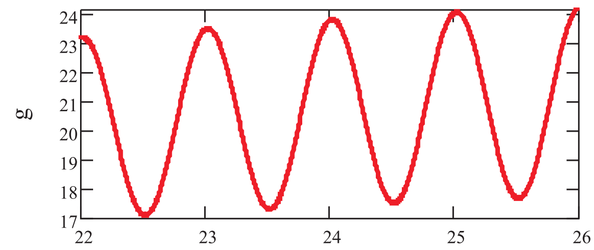

FIG. 8: The figure shows how the conductance depends on ν when the interface is disordered for parameters similar to used for the black curve in Fig.4, $e^{-\eta} \simeq 0.1$. Now the conductance behavior qualitatively agrees with the experimental data.

V. ACKNOWLEDGMENTS

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APPENDIX A: APPENDIX 1

For the sake of a reader's convenience, we present in this appendix the following theorem.[23,24] Let us Q be a 2×2 (complex) matrix with the determinant equal to unity then

$$Q^{n+1} = \begin{pmatrix} Q_{11}U_n(a) - U_{n-1}(a) & Q_{12}U_n(a) \\ Q_{21}U_n(a) & Q_{22}U_n(a) - U_{n-1}(a) \end{pmatrix}, \quad (A1)$$

where $a = \mathrm{Tr}\,Q/2$ and $U_n(a)$ is the Chebyshev polynomial of the second kind[25]

$$U_n(a) = \sin[(n+1)\arccos(a)]/\sqrt{1-a^2}.$$

APPENDIX B: DISORDER AVERAGING

In this appendix we present the details of derivation for Eqs. (41). Let us consider the matrix

$$\begin{pmatrix} A_{n+1} & B_{n+1} \\ C_{n+1} & D_{n+1} \end{pmatrix} = \left\langle M\Phi_{n+1} \begin{pmatrix} A_n & B_n \\ C_n & D_n \end{pmatrix} \Phi^\dagger_{n+1} M^\dagger \right\rangle \tag{B1}$$

with

$$\begin{pmatrix} A_0 & B_0 \\ C_0 & D_0 \end{pmatrix} = M\sigma_z M^\dagger. \tag{B2}$$

Then the quantity $R_{ee} - R_{he}$ in Eq. (33) is equal to A_{n-1}. It is convenient to introduce a vector $\Psi_n = (A_n, D_n, B_n, C_n)^T$ such that

$$\Psi_{n+1} = \mathbb{T}\left(e^{-\eta}\right)\Psi_n, \quad \mathbb{T}(x) = \begin{pmatrix} \tilde{S} & -xU \\ W & -xV \end{pmatrix} \tag{B3}$$

where the elements U, V and W of the transfer-matrix $\mathbb{T}$ are the following 2×2 matrices

$$W = \begin{pmatrix} r_{ee}r^\star_{he} & r_{eh}r^\star_{hh} \\ r_{he}r^\star_{ee} & r_{hh}r^\star_{eh} \end{pmatrix}, \tag{B4}$$

$$U = \begin{pmatrix} r_{ee}r^\star_{eh}e^{i2\pi\nu - i\overline{\delta\phi}} & r_{eh}r^\star_{ee}e^{-i2\pi\nu + i\overline{\delta\phi}} \\ r_{he}r^\star_{hh}e^{i2\pi\nu - i\overline{\delta\phi}} & r_{hh}r^\star_{he}e^{-i2\pi\nu + i\overline{\delta\phi}} \end{pmatrix}, \tag{B5}$$

$$V = \begin{pmatrix} r_{ee}r^\star_{hh}e^{i2\pi\nu - i\overline{\delta\phi}} & r_{eh}r^\star_{he}e^{-i2\pi\nu + i\overline{\delta\phi}} \\ r_{he}r^\star_{eh}e^{i2\pi\nu - i\overline{\delta\phi}} & r_{hh}r^\star_{ee}e^{-i2\pi\nu + i\overline{\delta\phi}} \end{pmatrix}. \tag{B6}$$

By using that $\Psi_0 = \mathbb{T}(0)\Psi_{-1}$ where $\Psi_{-1} = (1,-1,0,0)^T$, we find

$$\Psi_n = \mathbb{T}^n(x)\mathbb{T}(0)\Psi_{-1}. \tag{B7}$$

To the lowest order in $\exp(-\eta)$ we find from Eq. (B7)

$$A_n = \lim_{x\to 0} \mathrm{Tr}\begin{pmatrix} 1 & 0 \\ -1 & 0 \end{pmatrix}\Big[\tilde{S}^{n+1} - 2e^{-\eta}\,\mathrm{Re}\; e^{i(2\pi\nu - \phi)} \times \frac{\partial}{\partial x}(\tilde{S} + xZ)^n\Big] \tag{B8}$$

where

$$Z = \begin{pmatrix} r^2_{ee}r^\star_{eh}r^\star_{he} & |r_{eh}|^2 r^\star_{ee}r^\star_{hh} \\ |r_{he}|^2 r^\star_{hh}r_{ee} & r^{\star 2}_{hh}r_{eh}r_{he} \end{pmatrix}. \tag{B9}$$

1. Zero temperature $T = 0$

At vanishing temperature $T = 0$ the matrices $\tilde{S}$ and Z can be simplified drastically

$$\tilde{S} = \begin{pmatrix} |r_{ee}|^2 & |r_{eh}|^2 \\ |r_{eh}|^2 & |r_{ee}|^2 \end{pmatrix}, \quad Z = |r_{ee}|^2|r_{eh}|^2 e^{2i\theta}\begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}. \tag{B10}$$

With the help of the following identity

$$\begin{pmatrix} a & b \\ b & a \end{pmatrix} = \frac{\sigma_x - \sigma_z}{\sqrt{2}}\Big[a - b\sigma_z\Big]\frac{\sigma_x - \sigma_z}{\sqrt{2}} \tag{B11}$$

we obtain Eq. (41) as

$$A_{n-1} = R^n\left[1 + 4(n-1)e^{-\eta}|r_{ee}|^2|r_{eh}|^2 R^{-2}\cos 2\Omega\right]. \tag{B12}$$

2. Arbitrary temperature

At arbitrary temperature (energy) where is no special relations between r_{ab}. Then with the help of Eq. (A1) we find

$$A_n = A^{(0)}_n - 2e^{-\eta}\,\mathrm{Re}\; e^{i(2\pi\nu - \overline{\delta\phi})}A^{(1)}_n \tag{B13}$$

where

$$A^{(0)}_n = \det\nolimits_0^{n/2}\left[\frac{|r_{ee}|^2 - |r_{he}|^2}{\sqrt{\det_0}}U_{n-1}(a_0) - U_{n-2}(a_0)\right] \tag{B14}$$

and

$$\begin{aligned} A^{(1)}_n = {} & \det\nolimits_0^{\frac{n-1}{2}}U_{n-1}(a_0)(r^2_{ee}r^\star_{he}r^\star_{eh} - |r_{eh}|^2 r_{ee}r^\star_{hh}) \\ & -\det\nolimits_0^{\frac{n}{2}}\left[\alpha a_0 U'_{n-2}(a_0) + \frac{n}{2}\beta U_{n-2}(a_0)\right] \\ & +\det\nolimits_0^{\frac{n-1}{2}}\left[\frac{n-1}{2}U_{n-1}(a_0) - \alpha a_0 U'_{n-2}(a_0)\right](|r_{ee}|^2 - |r_{eh}|^2) \end{aligned} \tag{B15}$$

Here we have introduced the following notations

$$\det\nolimits_0 = |r_{ee}|^2|r_{hh}|^2 - |r_{eh}|^2||r_{he}|^2, \tag{B16}$$

$$a_0 = \frac{|r_{ee}|^2 + |r_{hh}|^2}{2\sqrt{\det_0}}. \tag{B17}$$

and

$$\beta = \det\nolimits_0^{-1}\Big(|r_{hh}|^2 r^2_{ee}r^\star_{eh}r^\star_{he} + |r_{ee}|^2 r^{\star 2}_{hh}r_{eh}r_{he} - 2|r_{eh}|^2|r_{he}|^2 r_{ee}r^\star_{hh}\Big), \tag{B18}$$

$$\alpha = \frac{r^2_{ee}r^\star_{eh}r^\star_{he} + r^{\star 2}_{hh}r_{eh}r_{he}}{|r_{ee}|^2 + |r_{hh}|^2} - \frac{\beta}{2}. \tag{B19}$$

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